ON THE BORDERS OF VAGUENESS AND THE VAGUENESS OF BORDERS

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Abstract. This article argues that resolutions to the sorites paradox offered by epistemic and supervaluation theories fail to adequately account for vagueness. After explaining the paradox, I examine the epistemic theory defended by Timothy Williamson and discuss objections to his semantic argument for vague terms having precise boundaries. I then consider Rosanna Keefe's supervaluationist approach and explain why it fails to accommodate the problem of higher-order vagueness. I conclude by discussing how fuzzy logic may hold the key to resolving the sorites paradox without positing indefensible borders to the correct application of vague terms.

Suppose in front of you is a heap of sand. You remove one grain and toss it aside. The collection is, of course, still a heap; a single grain surely could not make the difference between it being or not being one. So, you remove another grain, followed by another, and another, until eventually a single piece of sand—clearly, not a heap—remains where the heap once was. But how could this be, given you only removed a grain at a time? This problem takes the form of a sorites paradox and can be expressed more explicitly as follows:

> P. 1: A 1,000,000-grain collection of sand is a heap.
> P. 2: If a 1,000,000-grain collection is a heap, so is a 999,999-grain collection.
> P. 3: If a 999,999-grain collection is a heap, so is a 999,998-grain collection.
> …
> P. 1,000,000: If a 2-grain collection is a heap, so is a 1-grain collection.
> C.: A 1-grain collection of sand is a heap.

This argument is paradoxical since a seemingly unacceptable conclusion follows from apparently acceptable premises and reasoning.[1] Similar versions can be made using examples other than a

heap. Suppose there are a million patches of color ranging from blue to yellow across a long wall, with those on the far left being blue, those on the far right being yellow, and those in the middle being various shades of green. Each patch is visually indistinguishable from those adjacent to it. Starting with the leftmost patch, one would be forced to conclude that, if this is blue, then so is the patch to its immediate right; by extension, the yellow patch on the far right must also be deemed blue, which is patently false.

The problem elicited by these and other sorites paradoxes concerns the vagueness of language. Terms like "heap," "blue," "short," and "bald" are vague since they allow for borderline cases where, despite having all the information normally considered sufficient to determine whether the word may be used, it remains unclear whether or not to apply it.[2] We could count exactly how many grains are in a collection or measure the precise wavelength of a patch of color, yet still be unable to determine with confidence whether it is a heap or whether it is blue. Therefore, any solution to a sorites paradox must account for the nature of vagueness itself.

To resolve a paradox, one of four approaches may be taken: denying the paradox is logically possible, accepting the conclusion, claiming the reasoning is invalid, or denying the premises are all true.[3] For sorites paradoxes, the first option seems unviable since many forms of the paradox are clearly possible and can even be modeled in real life. The second option appears only slightly better since it implores us to accept that a single grain may be a heap. Peter Unger argues along these lines that due to widespread and inherent flaws in our language, there are in fact no such things as heaps: vague concepts are "incoherent."[4] He notes, however, that due to the unattractiveness of this solution, accepting the conclusion ought only to be a last resort if other approaches prove unfruitful.[5] The third strategy of attacking the reasoning seems unlikely to yield success since the argument relies exclusively on iterations of *modus ponens*. If all premises are true, it follows deductively that a 1-grain collection is a heap. This leaves only the fourth approach: denying the truth of at least one premise. Since the conclusion is absurd, yet the reasoning is valid, it looks to follow by *reductio ad absurdum* that at least one premise must be false.

Some who opt for this approach assert that despite contrary appearances, vague terms draw a precise border between their correct and incorrect application. Thus, one conditional premise of the form "if an *n*-grain collection is a heap, so is an *n*-1-grain collection" must be false since there is an exact minimum number of grains *n* required to make a heap.[6] Defenders of this so-called epistemic theory maintain that the number *n* is unknowable. We are necessarily ignorant of the borders drawn by vague predicates.[7] Chrysippus advanced this view in ancient Greece and the theory has been more recently developed by Roy Sorensen and Timothy Williamson.[8] Sorenson describes the boundaries of vague terms as an 'epistemic blindspot': a proposition which is wholly consistent yet inaccessible to us.[9] It is merely our ignorance to the limits of vague words' referents which gives the illusion of a paradox.

Critics might object that the claim we cannot know the boundaries to vague terms suggests these sharp boundaries do not exist in the first place.[10] Given sufficient time and effort, we would surely be able to figure out where the borders to vague terms lie—if they are real. It seems more probable, then, that vague terms do not draw mysterious precise boundaries, but rather the limits of their application are also vague.

However, this objection subtly endorses a verificationist theory of meaning, which is itself implausible.[11] If we look at a borderline collection of sand and state it is a heap, we have no way of checking whether we have applied the word correctly. By the verificationist theory of meaning, describing objects using vague language would be meaningless. This theory was popularized by A. J. Ayer and other logical positivists some decades ago, but has since been criticized for a plethora of reasons. Notably, W. V. O. Quine argued that the analytic-synthetic distinction central to verificationism appeals to the synonymy of different terms, which in turn requires that such terms be necessarily interchangeable.[12] But this invokes the notion of analyticity in order to define it, so the verificationist account is circular.[13] Additionally, Quine noted that we cannot empirically test claims in isolation, but only in conjunction with numerous background assumptions,[14] so a clear-cut verification condition often cannot be established for individual statements.[15] If the epistemic theory of vagueness is to be refuted on

these grounds, verificationism must first be resurrected from the philosophical graveyard.

Moreover, Williamson accounts for why we cannot know whether a vague term applies in borderline cases. He argues our limited cognitive and sensory abilities are necessary to gain knowledge about the world since knowledge requires being non-accidentally right.[16] If I believe I will be dealt a royal flush on my next poker hand, and this belief is subsequently proven true, I could not claim to have known I was going to receive those cards. Likewise, if I stated, "this is a heap" about a borderline collection with exactly *n* grains, I could not claim to have known that either. If the collection had slightly fewer grains and was therefore just below the heap threshold, my vision is not sufficiently precise to have noticed the difference. I would still have remarked "this is a heap," but this time I would have been wrong. Because knowledge requires being non-accidentally right, I could not claim to have known the collection was a heap in the first case: I was only right through luck. According to Williamson, we can only know a claim is true—be that "this is a heap," "he is bald," or "she is tall"—if such a claim would be true in all similar cases, that is, cases falling within a margin for error.[17] This excludes the possibility that we could know if we are using vague terms appropriately in borderline cases.

Williamson also explains how vague terms may generate sharp boundaries of reference. He asserts that the meaning of a word must be determined by how it is used in a language.[18] It certainly seems plausible that no specific combination of sounds or squiggles would have a meaning until a linguistic community began using the word to express a particular claim.[19] Consider the word "smartphone," which, just decades ago, had no accepted meaning since it had not been created and used by English speakers to refer to a type of technological device. The meaning of "smartphone" has changed over recent years too. Whereas being able to send emails and play Tetris qualified a cellular device to be a smartphone some years ago, nowadays more advanced features such as Bluetooth or GPS would be required. Thus, the meaning of "smartphone" is not completely fixed, but rather its definition has developed alongside our use of the term. As Williamson puts it, there is "no difference in meaning without a difference in use."[20]

But, how might we calculate meaning from use? For vague terms like "heap," perhaps the meaning could be determined by how "heap" has been used by speakers of English in the past. If a parent used the word "heap" to refer to their child's sandcastle of exactly 14,832 grains, this would promote the meaning of "heap" as including collections of this size, so long as other English speakers had not refrained from using the word in similar situations. The past use of a vague term may be what determines whether it applies correctly or incorrectly to a particular situation.

There may be times when we may confront a borderline case slightly different to all those experienced and described before. In such cases, we must think hypothetically and consider whether speakers of the language would tend more towards applying a vague term or not. This involves considerations of context—presumably, a collection of pumpkins requires fewer objects to qualify as a heap than does a collection of sand. Still, if in any given situation the majority of English speaking people would accept that the word "heap" may be used to describe a collection, then perhaps we could say that the meaning of "heap" would include that particular collection. There is, of course, no way of testing this in real life, but it remains a fact—albeit unknowable to us—whether the majority of speakers in a linguistic community would either agree or disagree with using a vague term to describe a borderline case.

However, this approach is too simplistic. Considering the vague word "thin," Williamson notes we may systematically misjudge people by their body size or clothing.[21] Most English speakers might classify somebody as thin when they are in fact wider than somebody else who would be considered not-thin. Meaning cannot just be reduced to statistics about whether or not people would apply a term. Williamson acknowledges we cannot know how exactly to derive meaning from use since "there is no algorithm for calculating the former from the latter."[22] He claims this is not a problem for the epistemic theory since we can still maintain that there exists a sharp border separating where vague words do and do not apply based on use; it is just unknown to us.

Williamson's argument fails since linguistic communities which supposedly determine meaning through their dispositions are not

themselves bounded with any precision. Timothy Endicott notes there are many borderline cases for who should be included in a speech community: speakers of other languages who know some English, children with partially developed vocabularies, or just certain individuals too eccentric in their word usage.[23] Even within a community, there will be variation in word usage corresponding to ethnic, social, and gender differences among speakers. If we consider these innumerable linguistic varieties as separate languages, it remains unclear which language is being used in any particular circumstance.[24] An individual may make a vague statement which is true in the language of their age group, but false in that of their town, yet true in that of their country overall. The dispositions of speakers from these various communities would together form slightly different compositions and hence draw the cut-off point for a vague term differently. The epistemic notion that sharp boundaries can be determined by the collective dispositions of a group rests on being able to decide which language community is the correct one for any given situation. But, as Endicott notes, there is no non-arbitrary way to establish which individuals should comprise a speech community.[25]

The epistemic solution to the sorites paradox appears unconvincing, but another method of rejecting the premises may hold more promise. Supervaluationism maintains that the meaning of vague terms is incomplete, so there are various plausible ways to draw precise boundaries to their scope.[26] Rosanna Keefe asserts that this theory provides an account of vagueness which "does vastly better than its rivals" at balancing intuitions and linguistic practices alongside theoretical considerations.[27] According to supervaluationism, borderline cases are confounding because vague terms do not contain sufficient meaning for us to know whether or not they apply.[28] Consider the word "short." There are many ways we could make the word (as it applies to an adult male) more precise, such as by claiming it refers to men below 5'4", or 5'6", or 5'8". Each of these proposed cut-off points appears a reasonable way of clarifying the meaning of "short." In supervaluationist terminology, this is called "sharpening" the term. Under each sharpening, it is still clear that a man who is 4'6" is short and a man who is 6'6" is not.

Likewise, we could sharpen "heap" to refer to collections of at least 7,103 grains of sand, or 11,518, or 20,973. Whichever is accepted, a 1,000,000-grain collection will be deemed a heap and a 1-grain collection will not since it is obvious on any sharpening whether the word heap does or does not apply in definite cases. Somewhere in the borderline cases, however, for any given sharpening, a conditional premise will not hold. Unlike the epistemic theory, which posits a single correct boundary, supervaluationism maintains there are many potential candidates, each drawing a different border between heap and non-heap.

Keefe argues vague statements are true if and only if they are true on every possible sharpening, false if and only if they are false on every possible sharpening, and neither true nor false otherwise.[29] This commits supervaluationism to acknowledging truth-value gaps—statements which are neither true nor false—since borderline cases for "heap," "blue," or "bald" will inevitably be true on some sharpenings yet false on others. However, Keefe notes that despite rejecting the principle of bivalence, supervaluationism still preserves most of classical logic since statements such as "either this book is yellow or it is not yellow" remain true, even though neither disjunct will be true if the book is a borderline color.[30] This appears to sidestep the difficulty faced by epistemicists of different individuals and speech communities generating different borders to a vague term's scope. By acknowledging that vague terms are incomplete in their meaning, supervaluationists need not assert a single mysterious boundary. In noting, that on any sharpening, there will come a point where a vague term ceases to apply, we resolve the sorites paradox by rejecting at least one premise.

However, there are several problems with supervaluationism. One of the theory's most serious shortcomings is its inability to account for higher-order vagueness.[31] Even if we accept there are many plausible ways to sharpen a vague term, the notion of sharpening is itself vague. We may still ask whether a collection of sand is *definitely* a heap or whether it falls within the range of borderline cases. [32] In other words, we may ask if a reasonable sharpening could be made which would exclude the collection of interest. Supervaluationism merely duplicates the problem faced by epistemicism, positing an unclear

border between a vague term and potential sharpenings, as well as a border between definitely or definitely not applying. One could, for instance, ask whether a man who is 5' in height is definitely short or whether a reasonable sharpening might be made to exclude those above 4'11". Allowing all possible sharpenings would make vague terms meaningless. If "short" were sharpened to include only those below 3' in height, this would be wholly inconsistent with our understanding and use of the word. Without providing conditions for what qualifies as an acceptable sharpening, supervaluationism fails to define vagueness convincingly.

If resolutions to the paradox based on rejecting the premises are unsuccessful, the next best approach would be to target the reasoning itself. At first glance, this seems near impossible: the first premise, "a 1,000,000-grain collection of sand is a heap," is clearly true, and subsequent premises merely repeat *modus ponens*; thus, the absurd conclusion appears an undeniably valid result. But perhaps the concept of truth could be adapted to accommodate vague statements. Classical logic permits just two truth-values: totally false and totally true, which may be represented as 0 and 1, respectively.[33] Fuzzy logic, on the other hand, allows for infinitely many truth-values between 0 and 1 to represent borderline cases that are neither definitely true nor false.[33] A statement with a truth-value of 0.95 would be almost but not entirely true. This could explain the paradox as arising due to many of the conditional premises being less than completely true.[34] As *modus ponens* is applied down the chain of reasoning, a certain "leakage" of truth occurs.[35] While the number of grains in a collection decreases, so does the truth-value of the claim that the collection is a heap. By the time the collection is sufficiently small to be considered definitely not a heap, the truth-value will have reduced to 0.

Fuzzy logic, I would argue, coincides better with common understandings of vagueness than other logical varieties permit. Suppose I celebrate finishing this paper with a twelve-pack of beer. After taking the first sip, I will not be drunk. But imagine I keep drinking steadily until I finish the last bottle. By then, I will definitely be drunk. Throughout my evening, however, there will be a period during which it is not clear whether or not I am. It seems reasonable that if somebody asked, "Are you drunk?" when I have finished six

beers, that I reply "Well, that's *somewhat* true." To respond with a definite "yes" or "no" seems inconsistent with the experience of becoming gradually intoxicated. Giving the claim "I am drunk" an intermediate truth-value between 0 and 1 seems to better represent real-life experience and natural language describing drunkenness than classical logic allows. Figure 1 illustrates this.

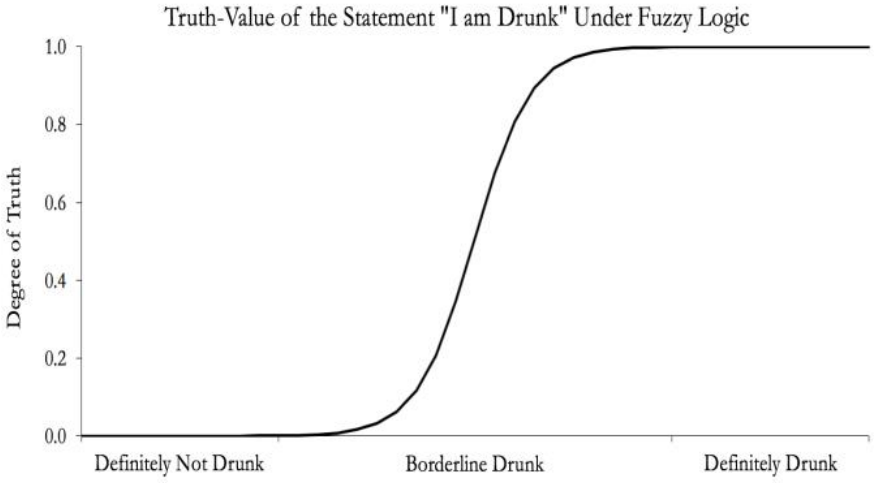

Fig. 1: Truth of vague statements according to fuzzy logic.

Several philosophers have noted that fuzzy logic may fare no better than supervaluationism at explaining higher-order vagueness.[36, 37] The sharp boundary between truth and falsity posited by the epistemic theory has been avoided, but we still may ask whether it is definitely true that somebody is drunk, or that a collection is a heap. Fuzzy logic seems to trade a single arbitrary border for one either side of the range of indeterminate truth-values, and hence fails to explain vagueness suitably.

Fuzzy logic can accommodate higher-order vagueness by assigning additional truth-values to statements so the borders between indeterminate and determinate truth-values are blurred. Suppose statement A is assigned a truth-value of 0.8 so that V(A) = 0.8. We might then ask, "What is the truth of the claim that V(A) = 0.8?". If it is completely true, we could express this as V(A) = (0.8, 1); if it is true to degree 0.9, we could write V(A) = (0.8, 0.9). This process can be extended so that the infinitely many levels of higher-order vagueness are accounted for.[38] The border between indeterminate truth and determinate truth could be smoothed over by going from V(A) = (1, 0.98) to V(A) = (1, 0.99), and likewise for third-order vagueness,

fourth-order vagueness, and so on *ad infinitum* by adding additional levels of truth-values. The implausibility of epistemic and supervaluation theories suggests that perhaps continuing to search for sharp boundaries to vague language is futile. Likewise, our inherent uncertainty when using words such as "heap" or "bald" to describe borderline cases may signify a lack of clear-cut limits to the correct application of vague terms. By extending our definition of truth to include values between 0 and 1 where appropriate, fuzzy logic may succeed in blurring the boundaries between truth and falsity without creating additional sharp ones, thus providing a satisfactory account of vagueness and resolution to the sorites paradox.

There are some limitations to this form of logic. For one, it is unclear how we could accurately assign indeterminate truth-values.[39] However, an inability to express and categorize truth-values accurately is hardly an objection to their existence; the truth of a statement is unrelated to our descriptive aptitudes.[40] If the meaning of vague terms is constructed by their common use, as Williamson suggests, it is still quite possible for an individual to apply these words incorrectly. One may mistakenly describe a woman wearing high-heeled shoes as tall or misjudge the degree to which a vague term such as red applies to a borderline-red strawberry. Even under classical logic, we may make incorrect judgments by asserting a true statement to be false or vice versa. For instances of vagueness, indeterminate truth-values could be estimated by comparing borderline cases with one another.[41] Given two borderline short men, the shorter of the pair can be assigned a higher truth-value for "is short" even though the truth-value for each is less than 1.

Fuzzy logic also appears to generate incorrect truth-values for certain expressions.[42] If somebody is borderline bald, the statement "He is bald and he is not bald" may have a truth-value between 0 and 1 under fuzzy logic. In many situations requiring logical analysis, however, only determinate truth-values of 0 and 1 are needed, so standard logical theorems can still be applied. Despite fuzzy logic having some problematic aspects, it appears to provide a notion of vagueness that is consistent with experience and natural language while managing to blur the boundaries between truth and falsity even at higher levels of vagueness. Consequently, it appears a more promising

method of resolving the sorites paradox than epistemic or supervaluationist approaches.

Williamson's epistemic theory posits a sharp boundary between the correct and incorrect application of vague terms. However, language is not used in a way that could determine sharp borders to a vague word's meaning. The supervaluationist theory that Keefe defends accommodates the plurality of ways to draw a line between truth and falsity, but the invocation of a single indeterminate truth-value fails to account for higher-order vagueness. Fuzzy logic resolves the sorites paradox by claiming the reasoning is flawed: the conditional premises "leak" truth, which eventually generates a false conclusion. Though often thought to be susceptible to higher-order vagueness, fuzzy logic can blur the boundaries between determinate and indeterminate truth such that higher-order vagueness is taken into account. Vague terms, then, appear not to have precise borders to their sphere of reference. Instead, the borders are themselves vague—or at least fuzzy.[43]

Notes

[1] R. M. Sainsbury, *Paradoxes*, 3rd ed. (Cambridge: Cambridge University Press, 2009), 1.

[2] Sainsbury, *Paradoxes*, 41.

[3] Dominic Hyde, "Sorites Paradox," in *The Stanford Encyclopedia of Philosophy*, Winter 2014 ed., ed. Edward N. Zalta (Stanford University, 2014). https://plato.stanford.edu/archives/win2014/entries/sorites-paradox/.

[4] Peter Unger, "There Are No Ordinary Things," *Synthese 41*, no. 2 (1979): 118, http://www.jstor.org/stable/20115446.

[5] Sainsbury, *Paradoxes*, 49.

[6] Likewise, there is an exact maximum number of hairs required to be bald, a minimum height to be tall, an exact cut-off point for a color to be blue, and so on for other possible sorites paradoxes.

[7] Sainsbury, *Paradoxes*, 50.

[8] Rosanna Keefe, *Theories of Vagueness*, (Cambridge: Cambridge University Press, 2000), ProQuest Ebook, 62.

[9] Roy Sorensen, "Contagious Blindspots: Formal Ignorance Spreads to Peers," *American Philosophical Quarterly 52*, no. 4 (2015): 335, http://apq.press.illinois.edu/view.php?vol=52&iss=4&f=sorensen.pdf.

[10] Sainsbury, *Paradoxes*, 50.

[11] Sainsbury, *Paradoxes*, 50.

[12] W. V. O. Quine, "Main Trends in Recent Philosophy: Two Dogmas of Empiricism," *Philosophical Review 60*, no. 1 (1951): 29, http://www.jstor.org/stable/2181906.
[13] Quine, "Main Trends in Recent Philosophy,"34.
[14] Quine, "Main Trends in Recent Philosophy," 38.
[14] When testing the melting point of a newly discovered substance, say, it would be necessary to assume that the thermometer is working, that the substance has not been tainted, and so on.
[15]Timothy Williamson, *Vagueness*, (New York: Routledge, 1994), ProQuest Ebook, 217.
[16] Timothy Williamson, *Vagueness*, (New York: Routledge, 1994), ProQuest Ebook, 226.
[17] Williamson, *Vagueness*, 206.
[18] Anat Biletzki and Anat Matar, "Ludwig Wittgenstein," in *The Stanford Encyclopedia of Philosophy*, Fall 2016 ed., ed. Edward N. Zalta (Stanford University, 2016). https://plato.stanford.edu/archives/fall2016/entries/wittgenstein/.
[19] Williamson, *Vagueness*, 206.
[20] Williamson, *Vagueness*, 207.
[21] Williamson, *Vagueness*, 206.
[22] Timothy Endicott, *Vagueness in Law*, (New York: Oxford University Press, 2000), 109.
[23] Endicott, *Vagueness in Law*,108.
[24] Endicott, *Vagueness in Law*, 108.
[25] Linda Claire Burns, *Vagueness: An Investigation into Natural Languages and the Sorites Paradox*, (Dordrecht, The Netherlands: Kluwer Academic Publishers, 1991), 66.
[26] Rosanna Keefe, "Vagueness: Supervaluationism," *Philosophy Compass 3*, no. 2 (2008): 316, doi:10.1111/j.1747-9991.2008.00124., 152.
[27] Sainsbury, *Paradoxes*, 51.
[28] Keefe, "Vagueness: Supervaluationism," x.
[29] Keefe, "Vagueness: Supervaluationism," x.
[30] Sainsbury, *Paradoxes*, 55.
[31] Sainsbury, *Paradoxes*, 55.
[32] Sainsbury, *Paradoxes*, 57.
[33] Petr Cintula, Christian G. Fermüller, and Carles Noguera, "Fuzzy Logic," in *The Stanford Encyclopedia of Philosophy*, Fall 2017 ed., ed. Edward N. Zalta (Stanford University, 2017). https://plato.stanford.edu/archives/fall2017/entries/logic-fuzzy/.
[34] Cintula, Fermüller, and Noguera, "Fuzzy Logic"
[35] Sainsbury, *Paradoxes*, 58.
[36] Sainsbury, *Paradoxes*, 62.
[37] John MacFarlane, "Fuzzy Epistemicism," in *Cuts and Clouds: Vagueness, its Nature, and its Logic*, ed. Richard Dietz and Sebastiano Moruzzi (Oxford: Oxford University Press, 2010), ProQuest Ebook, 455.
[38] Jack Copeland, "Indeterminacy" (lecture, University of Canterbury, Christchurch, New Zealand, October 9, 2017).

[39] R. M. Sainsbury, *Paradoxes*, 3rd ed. (Cambridge: Cambridge University Press, 2009), 61.
[40] Jack Copeland, "Indeterminacy" (lecture, University of Canterbury, Christchurch, New Zealand, October 9, 2017).
[41] Sainsbury, *Paradoxes*, 61.
[42] Roy Sorenson, "Vagueness," in *The Stanford Encyclopedia of Philosophy*, Winter 2016 ed., ed. Edward N. Zalta (Stanford University, 2017). https://plato.stanford.edu/archives/win2016/entries/vagueness/.
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